Binomial Distributions Binomial Random Variables

10 ?'s

4 conditions:

- Must have a set # of trials/observations $n=10$

0.25

- All of the observations are independent of each other

2, 3

- Only 2 outcomes: success/failure

1 success

3 failure

- The probability of success remains constant $(p=0.25)$

Notation: $B(n,p)$ $B(10, 0.25)$

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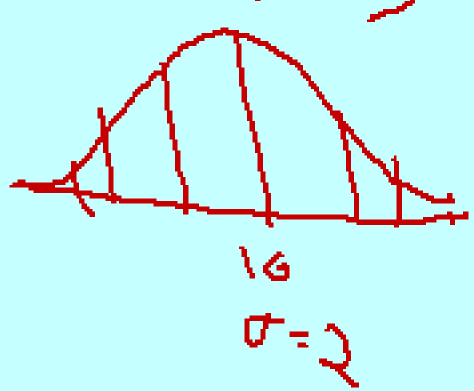
$\mu_x = n \cdot p$

$\sigma_x = \sqrt{n \cdot p \cdot (1-p)}$

$N(10, 2)$



Binomial Probabilities

- **We already know the formula for this!**
- **Binomial random variables are just…** discrete random variables
- **Formula:** $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$
- **Example: Computer chips have a 25% chance of being defective. Create the probability distribution for X, if X is the # of defective chips in a sample of 4.**

X	P(X)	
0	$(4nCr0)(0.25^0)(0.75^4)$	0.3164
1	$(4nCr1)(0.25^1)(0.75^3)$	0.4219
2		0.2109
3		0.0469
4		0.0039

So let's answer some easy questions:

P(X=2) = 0.2109

P(X<2) = 0.7383

P(X≥3) = 0.0508

P(2≤X≤4) = 0.2617

Now let's look at changing the sample size to 10, and answering similar questions: B(10, 0.25)

X	P(X)
0	0.0563
1	0.1877
2	0.2816
3	0.2503
4	0.1460
5	0.0584
6	0.0162
7	0.0031
8	0.00039
9	0.000029
10	0.000000954

P(X=9) = 0.000029

P(X<4) = 0.7759

P(X ≥ 6) = 0.01972

P(5 ≤ X ≤ 7) = 0.0777

Would you want to answer these questions for a sample size of 30? Of 50? Of 100?

So we can use the calculator

using formula

$P(12 \le X \le 20)$

For P(X=k)

- Use binompdf(n, p, k)

- k = # of "successes"

- pdf = probability density fctn.

x	P(x)

For $P(X \leq k)$

- Use binomcdf(n, p, k)
- k = # of successes
- Notice that is ONLY GIVES YOU $\leq$
- cdf = cumulative density fctn.

However you MUST still write Prob. notation

$P(X \gtrless k) = \text{answer}$

$P(X \leq 10)$

$P(X < 15) = P(X \leq 14)$

$P(X \geq 12) = 1 - P(X \leq 11)$

$P(X > 9) = 1 - P(X \leq 9)$

Example:

- John is taking archery.
- He has a 30% chance of hitting the target each time he shoots.
- He shoots 8 times

$B(8, 0.30)$

1) What is the probability that he hits the target 4 times?

$P(X=4) = \text{binompdf}(8, 0.3, 4) = 0.1361$

2) What is the probability that he hits the target 2 times or less?

$P(x \leq 2) = \text{binomcdf}(8, 0.3, 2) = 0.5518$

3) What is the probability that he hits the target at least 3 times?

$P(x \geq 3) = 1 - P(x \leq 2) = 1 - \text{binomcdf}(8, 0.3, 2) = 0.4482$

4) What is the probability that he hits the target less than 5 times?

$P(x < 5) = P(x \leq 4) = \text{binomcdf}(8, 0.3, 4) = 0.9420$

5) What is the probability that he hits the target more than 6 times?

$P(x > 6) = 1 - P(x \leq 6) = 1 - \text{binomcdf}(8, 0.3, 6) = 0.0013$

Try this example on your own:

150 businesses are sent mailings asking them to answer a survey question and send the mailing back. The probability of nonresponse is 55%.

1) **What is the average number of businesses that WILL respond?** $\mu_x = 67.5$

2) **What is the standard deviation of the number of businesses that WILL respond?** $\sigma_x = 6.093$

The rest of the questions deal with the binomial random variable X, where X is the number of businesses that WILL respond. Don't forget your prob. notation!

3) **What is the probability that 75 businesses will respond?** $P(x=75)=0.0306$

4) **What is the probability that 60 businesses or less will respond?** $P(x \le 60) = 0.1251$

5) **What is the probability that 60 businesses or more will respond?** $P(x \ge 60) = 1 - P(x \le 59) = 0.9058$

6) **What is the probability that less than 60 businesses will respond?** $P(x < 60) = P(x \le 59) = 0.0942$

7) **What is the probability that greater than 60 businesses will respond?** $P(x > 60) = 1 - P(x \le 60) = 0.8749$

8) **What number of surveys would you have to send out if you wanted to be able to expect to get 90 back?**

$90 = n \cdot p$

$90 = n \cdot 0.45$

$n = 200$

Try worksheet 5.1- Binomial Distributions